Location and allocation decisions with uncertain demand in a two-echelon supply chain using ant algorithm

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Summary

Industries face several challenges in making the location-allocation decisions for their new factories to maintain their competitiveness in the market. Those challenges include the facility location decision, its distribution assignment through entire supply chain, and its distribution quantity. The decisions become more challenging because the customer demands are always uncertain, further, the optimization-based methods can only solve the small-scaled problems. The location-allocation decision can give the method in deciding complex problem which is where to place the new facilities. This paper presents a bi-level stochastic programming model to represent the location-allocation decisions in two-echelon supply chain, then the ant algorithm is proposed to solve it. The objective is to maximize the profit of the manufacturing firms. The contribution of the study focuses on a representation of system configuration design for a two-echelon supply chain, where we solve the location-allocation problem by locating a number of factory in a finite set of potential sites, then assigning task between factories and marketplaces.

Keywords: location-allocation problem, ant algorithm, stochastic programming, two-echelon supply chain
1. Introduction

Industries face several challenges in making the location-allocation decisions which is need to be determined simultaneously for their new factories to maintain their competitiveness in the market. They have to debit capital to build newer generation of factories, however since the customer demands are uncertain, they suffer from both investment and the market risks at the same time. Those challenges include the facility location decision, its distribution assignment through entire supply chain, and its distribution quantity. In order to yield tremendous profit and productivity in the entire SC and individual firms, there should be an effective and efficient location and allocation strategy formed by the synergy of manufacturers and marketplaces. Further, the optimization-based methods can only solve the small-scaled problems, thus we propose to use a metaheuristic methods to solve the stochastic problem and to reduce the computational time of determining the optimal solution. This study formulates a location-allocation problem against uncertain demand in a two-echelon supply chain network by developing a model representing the problem to locate a number of factories among a finite set of potential sites and allocating the task assignment between factories and marketplaces to maximize profit. This study also proposes ant algorithm, one of a powerful metaheuristic methods to find the optimal solution of the model.

The rest of the paper is organized as follows. Section 2 reviews the related research on location-allocation problems and ant algorithms. Section 3 explains the problem formulation and develops the models for the problem. Section 4 discusses the proposed algorithm. Section 5 draws the conclusion of this study.

2. Literature Review

This research draws on literature in three areas. The first is the facility location allocation problem, the second is the two-stage programming, the third is the metaheuristic methods. Researchers have made progress on research to solve location and allocation problems, Revelle and Laporte (1996) reviewed literature regarding location problems and described formal statements of the problems. Santoso et al. (2005) proposed a stochastic programming model and a solution algorithm to solve supply chain network design problems, while Bischoff and Dächert (2009) compared various traditional and new search methods in a generalized class of location-allocation problems.

Further, two-stage programming is a special case in multi-stage programming commonly used to solve the facility location and allocation problems (Schütz et al. 2009, Xu and Nozick 2009). Laporte et al. (1994) considered a class of capacitated facility
location problems where have stochastic customer demands by formulating it as a stochastic two-stage integer linear program. Wang et al. (2011) addressed the location and allocation decisions in a two-echelon supply chain with stochastic demand by a genetic-algorithm based solution. However, past research indicates that comprehensive consideration of industrial properties in an SC has not yet been fully incorporated in a formal representation. Moreover the addressed location-allocation problem is highly complex. Melo et al. (2009) gave a comprehensive literature review of facility location models in supply chain management context. They found that the literature integrating uncertainty in SCM with location decisions is still scarce, only very few papers address stochastic parameters combined with multi-layer network structure.

Optimization-based approaches and exact solution methods were developed by many researchers for solving such problems, but those approaches are inefficient in a finite-time context. One of the other option for solving it is using soft-computing, but its applications remain scarce. Some researchers thus have developed a variety of heuristic and soft-computing methods. Some metaheuristic methods, such as Particle Swarm Optimization, Genetic Algorithm, Ant Algorithm, Simulated Annealing, and Tabu Search had been used by researchers for solving the location-allocation problem (Shankar, et al. 2013, Wang, et al. 2011, Zhou, et al. 2003, Albareda-Sambola, et al. 2009, Loranca, et al 2014). Ant algorithms is one of them which is recently developed, population-based, heuristic algorithms to deal with highly complex and dynamic problems. Many researchers are inspired by ant colony optimization (ACO) first introduced as the ant colony system (ACS) (Dorigo et al., 1999). In several industrial situations, ACO algorithms have been shown to offer preliminary success for some problems, but need to be adjusted to meet specific needs in given industry. For instance, Ren and Awasthi (2015) present four metaheuristics based solution approaches to address the capacitated location allocation problem on logistics network. They concludes that ACO showed better performance over others. But the research did not consider the stochastic demand and there is only one decision and one stage on its model. Moreover, Arnaout (2013) addresses the Euclidean location-allocation problem with an unknown number of facilities using ant colony optimization algorithm. Experiment results show that ACO outperformed GA and reach better solutions in a faster computational time. But, the objective function of that research is minimizing the total costs, thus do not consider the revenue and profit.

3. Problem formulation

The addressed capacitated location-allocation problem is represented in a stochastic mixed integer linear programming (MILP) model. The two-echelon SC model is
composed of factories and marketplaces, where the factories determine which markets to fulfill. A factory can serve several marketplaces, while each marketplace can be served only by one factory. There are several assumptions made for the model, the model is for single period and single product and factories are allowed to not satisfy all customer demands. The notations of the model are derived as follows.

The notations used in the model are:

- $i$: Number of factories ($i = 1, 2, \ldots, n$)
- $j$: Number of marketplaces ($j = 1, 2, \ldots, m$)
- $a$: Number of demand scenarios ($a = 1, 2, \ldots, N$)
- $f_i$: the fixed cost of setting up a facility at site $i$
- $c_{ij}$: the channel/distribution cost from factory $i$ to marketplace $j$
- $\xi^a$: a set of demand of all the marketplaces, in scenario $a$, $\xi^a = \{\xi^a_1, \xi^a_2, \ldots, \xi^a_m\}$ where $\xi^a_j$ is the customer demand in marketplace $j$, in scenario $a$.
- $p_{ij}$: the profit per unit distributed from factory $i$ to marketplace $j$
- $C_i$: the capacity of factory located at site $i$

The decision variables are:

- $x_i$: the decision to set up a factory, $x_i \in \{0, 1\}$ where $x_i = 1$ if a factory is set up at site $i$, $x_i = 0$ otherwise
- $y_{ij}$: configuration relationship between factory and marketplace, $y_{ij} \in \{0, 1\}$ where $y_{ij} = 1$ if factory $i$ distributes products to fulfill the demand to marketplace $j$, $y_{ij} = 0$ otherwise
- $w_{ij}(\xi)$: the quantity of products distributed from factory $i$ to marketplace $j$ in state of the world (demand scenario) $\xi$

The MILP model for two-echelon SC is in two level, presented as follows:

First level:

$$\min_{x, y} \hat{f}_N(x, y) = \sum_{i=1}^n f_i x_i + \sum_{i=1}^n \sum_{j=1}^m c_{ij} y_{ij} - E_{\xi} W(y, \xi)$$

Subject to

1. $y_{ij} \leq x_i \quad i = \{1, 2, \ldots, n\}, j = \{1, 2, \ldots, m\}$
2. $\sum_{i=1}^n y_{ij} \leq 1 \quad i = \{1, 2, \ldots, n\}, j = \{1, 2, \ldots, m\}$

Second level:

$$W(y, \xi) = \max \sum_{i=1}^n \sum_{j=1}^m p_{ij} w_{ij}(\xi)$$

Subject to

1. $\sum_{j=1}^m w_{ij}(\xi) \leq C_i \quad i = \{1, 2, \ldots, n\}$
2. $0 \leq w_{ij}(\xi) \leq \xi_j y_{ij} \quad i = \{1, 2, \ldots, n\}, j = \{1, 2, \ldots, m\}$

The objective function (1) maximizes total profit of $N$ scenarios (in a negative form for minimization). It is the sum of the factory setup cost and channel/distribution cost subtracted from the gross profit. $E_{\xi}$ is the mathematical expectation of gross profit with respect to demand scenarios $\xi$. Constraint (2) confines the relation between $y_{ij}$ and $x_i$. 
implying that a factory can serve many marketplaces. Constraint (3) specifies that a marketplace can be served by only one factory. For each demand scenario, the objective function (4) maximizes its gross profit. Constraint (5) confines the capacity limitation of factory $i$. Constraint (6) specifies that it is not necessary for factory $i$ to satisfy customer $j$’s demand.

4. Discussion

As mentioned before, there are several metaheuristic approach that can be used for solving the NP-hard problem like the complex location-allocation problem in the supply chain. In this part, we would like to tell about the differences of 4 methods and why we chose the ant algorithm as the proposed algorithm to solve the model. The first method is Simulated Annealing, it already had convergence proof based on the fact that behavior of simulated annealing can be modeled using Markov chains (Coello, et al, 2002). But, the computational time required by this model grow exponentially with respect to the size of the problem, the times spent will be more as the iterations grow. The second methods is Tabu search, where it allows movements to positions that may not be seem favorable from the current state. But it has difficulties of performing neighborhood movements in continuous search spaces (Coello, et al, 2002). Thus this make the search spaces are not diverse. The third method is particle swarm optimization, it is very simple in concept and in the implementation, easy and efficient to use, but it has difficulties to control the diversity for the multiobjective optimization. The fourth and last method is the ant algorithms, it has disadvantages in having diversity produced by the sophisticated selection mechanism with random action. The changes proposed by the negotiation mechanism of the algorithm are done in the decision variable space, not in the objective function space (Coello, et al, 2002). It makes the ant algorithm more suitable for solving the complex problems. One of main the disadvantages of ant algorithm is that the algorithm needs the process of decision making to do the ordering in the objective functions, requires parameters tuning, and needs the heuristic function to run the algorithms.

Based on the comparison, it can be seen that each methods has its own advantages and disadvantages, which are influencing the algorithm implementation and results. Thus we can’t choose the best methods, instead we can choose the methods that is suitable for the addressed problem. The purpose of the addressed location-allocation problem is to maximize profit while using less computational time. Therefore, we will not use the simulated annealing because of its growing computational time as the iterations running. We think that the ant algorithm is the most suitable methods for solving the complex
problems. Moreover, its disadvantages can be tailored by improving the algorithm to support the basic ant algorithm. The parameters can be tuned to find the best parameter combination, and the heuristic function can be designed based on the problems being addressed.

The algorithm used in this study is ant algorithm which is first introduced by Dorigo et al (1991). It is inspired by the ants’ behavior in searching the shortest past to find foods. The ant colony algorithm can be described in a multilayer network structure, each layer represents the possible path the ants can choose for reaching the food source, which is here the food is the final decision represents the decision whether to build a factory or not, the decision of allocation between the factories and marketplaces, also their allocation amount. The notations that will be used in the ant algorithm is shown below.

The notations:

- $k$: The number of the ant in generation.
- $i$: The node where the ant stays in the current state.
- $u$: The node where the ant will go in the next state.
- $j$: The decision of the ant.
- $q$: A random number uniformly distributed in $[0, 1]$.
- $J$: A node which was selected according to random-proportional rule.
- $\tau(i,u)$: The pheromone amount between node $i$ and $u$.
- $\eta(i,u)$: The heuristic desirability between node $i$ and $u$.
- $S_k(i)$: All of the nodes that can be chosen by ant $k$ while it is at node $i$ in the current state.
- $q_0$: A parameter whose range is from 0 to 1, and it determines the relative importance of exploitation versus exploration.
- $\alpha, \beta$: Parameters that determine the relative importance of pheromone versus heuristic value. $\alpha$ is the power of pheromone, $\tau(i,u)$, while $\beta$ is the power of heuristic value, $\eta(i,u)$.
- $\rho_l$: The local pheromone evaporating parameter.
- $\rho_g$: The global pheromone evaporating parameter.
- $L_b$: The best objective value of a generation.

When building a solution in Ant Colony, an ant $k$ at the current variable $i$ chooses the next variable $j$ to move to by applying a state transition rule defined in Eq. (7) (Dorigo and Gambardella, 1997).

$$j = \begin{cases} \arg \max_{u \in S_k(i)} \left\{ \left[ \tau(i,u) \right]^\alpha \left[ \eta(i,u) \right]^\beta \right\} & \text{if } q < q_0 \\ J & \text{otherwise} \end{cases}$$

(7)
\[ J = p_k(i,u) = \begin{cases} \frac{[\tau(i,u)]^\alpha[\eta(i,u)]^\beta}{\sum_{u \in S_k(i)}[\tau(i,u)]^\alpha[\eta(i,u)]^\beta} & \text{if } j \in S_k(i) \\ 0 & \text{otherwise} \end{cases} \]  

(8)

The state transition rule resulting from Eq. (7) and (8) tends to select the variable with the larger heuristic value and tends to be connected by edges with greater levels of pheromone. Each time an ant in variable \( i \) selects a variable \( j \) to move to, the ant in effect samples a random number \( q \). If \( q \leq q_0 \), then the ant chooses the variable corresponding to the edge with maximum \( [\tau(i,u)]^\alpha[\eta(i,u)]^\beta \) (exploitation); otherwise, the ant chooses a variable according to Eq. (8) (exploration).

An ant will decrease its the pheromone level on its visited edges by applying the following local updating rule in Eq. (9):

\[ \tau(i,j) = (1 - \rho_l).\tau(i,j) \]  

(9)

where \( \tau_0 \) is the initial pheromone level and \( \rho_l \) (\( 0 < \rho_l < 1 \)) is the local pheromone evaporating parameter. Each time an ant builds a solution, the local updating rule will lower both its visited edges’ pheromone and the edges’ attractiveness. As a result, ants will favor the exploration of edges not yet visited and prevent themselves converging on a common solution.

The global updating rule is applied after all ants have completed their journey, so the next ants can find a better solution. The global updating serves to provide a greater amount of pheromone to the edges of the solution with the best objective value. Hence, only the ant that found the best answer in the current generation is permitted to deposit pheromone. The pheromone level is modified according to Eq. (10):

\[ \tau(i,j) = (1 - \rho_g).\tau(i,j) + \Delta \tau(i,j) \]  

(10)

where

\[ \Delta \tau(i,j) = \begin{cases} (L_b)^{-1} & \text{if } (i,j) \in \text{ best solution} \\ 0 & \text{otherwise} \end{cases} \]  

(11)

In Eq. (10), \( \rho_g \) (\( 0 < \rho_g < 1 \)) is the pheromone evaporating parameter of global updating, and in Eq. (11), \( L_b \) is the objective function value of the best solution up to the current generation.

This study develops ant algorithm to make decision about which factories should be set up, which marketplace should be served by which factory, and its distribution quantities. The algorithm takes advantage of both ant searching strategy and greedy heuristics to find optimal solution. The ant algorithm procedure can be described in a pseudo code presented in Figure 1.
**Procedure: Ant Algorithm**

For generation = 1 to max_generation

For ant = 1 to m

Select probabilistically the next city according to

**exploration and exploitation mechanism**

Each ant builds a solution by **choosing the value of** $x_i$ and $y_{ij}$ with **local update**

End for

Apply **global trail updating rule** by using the best ant

End for

Figure 1 Procedure of the proposed Ant algorithm

As indicated in Figure 1, the algorithm starts when an ant goes out from the nest, and chooses the value of variables $x_i$ and $y_{ij}$ in sequence. Exploration and exploitation mechanisms are used while an ant chooses the trails, then a local trail updating rule is applied. When an ant finishes its journey, we will obtain a set of decision variables of $x_i$ and $y_{ij}$. If all the ants have completed their assignments, a generation is over and we will get $Q$ sets of decision variables and objective value where $Q$ is the number of ant in a colony. Global trail updating rule is applied by using the best solution, and the next generation is triggered after the global trail updating. The proposed algorithm stops when the maximal number of generation is reached.

As the discussion in the previous part suggested, there are needs to tune the parameters in order to fine the best parameter combination. Therefore we designed experimental setup by a 2-level fractional design for the parameters where each parameter has 2 level. The parameter being tuned are weight of heuristic value, $\beta$, value of trail which shows the best objective value for a generation, $\Delta \tau$, pheromone evaporating parameter of local updating, $\rho_l$, pheromone evaporating parameter of global updating, $\rho_g$, and relative importance of exploitation versus exploration $q_0$. There are 5 parameters used, each parameter has 2 levels so there are total $2^5 = 32$ combinations for the 2-level fractional design. Further as the discussion also suggested, there should be a developed for designing the heuristic value in the algorithm. Thus we propose to design the heuristics for determining the binary decision to build a factory, $x_i$, the decision of distribution allocation between factories and marketplaces, $y_{ij}$, and the quantity of products delivered by factory $i$ to marketplace $j$. 


5. Conclusion and Future Enhancement

This study has developed a two-level MILP model for the capacitated location-allocation problem in two-echelon supply chain network under uncertain demands. The decision of building factories among a finite set of potential sites, the decision of allocation between factories and marketplaces, also the distribution quantity have been determined in the model. This study contribute to use ant algorithm with greedy heursitics, one of metaheuristic approach than optimization-based and exact solution approach to solve a complex location-allocation problem in two-echelon SC. We discussed the options of metaheuristic methods that is the most suitable for the addressed problem, then we chose the ant algorithm despite its disadvantages. Further we develop the ant algorithm for solving the addressed problem and design a 2-level fractional design to do a parameter tuning for the algorithm in order to find the best combination of the parameters. In conclusion, ant algorithm with a designed heuristic value is developed to solve the complex location-allocation problem in two-echelon supply chain. For the future works, it is worthwhile to run the proposed algorithm for the addressed problem in order to test its efficiency of generating optimal solutions.

References


